

**STUDENT INTERNSHIPS AT THE MATHEMATICAL INSTITUTE OF THE
SERBIAN ACADEMY OF SCIENCES AND ARTS
2026.**



Development of algorithms for optimization problems

Mentors:

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Description: This topic belongs to the Operations Research scientific discipline. The work would be focused on the development of mathematical models, exact and heuristic optimization methods for various, academic and practical optimization problems (graph optimization, scheduling, transportation, location, newer variants of the traveling salesman problem, etc.). In addition to the application of various general-purpose exact methods (e.g., CPLEX, Gurobi, LINGO), exact and heuristic algorithms the explore a priori knowledge about the problem under consideration would be developed. Special attention would be paid to metaheuristics, general-purpose approximate methods that allow to overcome the shortcomings of exact methods, i.e., high demands on time and memory resources. The metaheuristics developed by Serbian researchers, Variable Neighborhood Search (VNS) and Bee Colony Optimization (BCO), would be specially promoted. Research would also be directed towards hybridizing metaheuristics with existing and newly developed machine learning techniques. In addition to development and implementation, work would be done on parallelization, theoretical and empirical analysis of metaheuristics. A significant segment of research within the summer internship would be focused on the implementation of optimization methods for real optimization problems that occur in science and industry. Through this work, students are enabled to develop and apply skills and techniques related to programming, optimization, graph theory and machine learning. Interested students will be able to apply data science techniques, such as visualization, data analysis, game theory, reinforcement learning, collaborative learning, and recommender systems.

Prerequisites: programming skills, C(C++), C#, Java, R, Python. Also, we suggest students to familiarize themselves with existing software for finding optimal parameter values of metaheuristic methods (eg iRace).

Relevant references:

1 IBM ILOG CPLEX 12.7 Optimization Studio CPLEX User's Manual, 2017.

https://www.ibm.com/support/knowledgecenter/SSSA5P_12.7.1/ilog.odms.studio.help/pdf/usrcplex.pdf

2 LINGO - The Modeling Language and Optimizer, LINDO Systems Inc. 2020.

<https://www.lindo.com/downloads/PDF/LINGO.pdf>

3 Gurobi Optimizer Reference Manual, Version 9/0, Gurobi Optimization LLC, 2020.

https://www.gurobi.com/wp-content/plugins/hd_documentations/documentation/9.0/refman.pdf

4 Woeginger, Gerhard J. “Exact algorithms for NP-hard problems: A survey”. *Combinatorial optimization—eureka, you shrink!*. Springer, Berlin, Heidelberg, 2003. 185–207.

5 Talbi, El-Ghazali, *Metaheuristics: from design to implementation*, John Wiley and Sons, 2009.

6 Hansen, Pierre, et al., Variable neighborhood search: basics and variants, *EURO Journal on Computational Optimization* 5(3):423-454, 2017.

7 Davidović, Tatjana, *Bee Colony Optimization: Recent Developments and Applications*, (plenary talk), Proc. Balkan Conference on Operational Research, BALCOR 2015, Constanta, Romania, Sept. 9-12, 2015. *Mircea cel Batran Naval Academy Scientific Bulletin*, 18(2):225-235, 2015.

8 Manuel López-Ibáñez, Leslie Pérez Cáceres, Jérémie Dubois-Lacoste, Thomas Stützle and Mauro Birattari. *The irace Package: User Guide*. IRIDIA, CoDE, Université Libre de Bruxelles, Brussels, Belgium, 2020. <https://cran.r-project.org/web/packages/irace/vignettes/irace-package.pdf> (verzija na srpskom jeziku: <http://www.mi.sanu.ac.rs/~tanjad/UputstvoZaIRace.pdf>)

Polydendriform algebras on hypergraph polytopes

Mentor: Jovana Obradović, jovana@mi.sanu.ac.rs

Description: The subject of this internship lies at the intersection of algebraic combinatorics, convex geometry and operad theory.

In 1998, Loday and Ronco introduced a seminal Hopf algebra on the linear span of rooted planar binary trees, which is deeply connected to the Malvenuto-Reutenauer Hopf algebra on permutations. Geometrically, planar binary trees and permutations label the vertices of two classical families of polytopes: associahedra and permutohedra. Loday-Ronco and Burgunder-Ronco later extended the associative products of these Hopf algebras to encompass all higher-dimensional faces of these polytopes, labeled by planar trees and surjections, respectively.

More precisely, Loday-Ronco introduced an associative product $*$ on planar trees as a shuffle of trees, where the shuffle $T * S$ of trees T and S is defined as a formal sum of trees whose nodes originate either from T , or from S , or from merging a node of S with a node of T . Loday and Ronco remarked that it is possible to split this product $*$ according to where the roots of the resulting trees originate from, giving rise to three operations “ \prec ”, “ \succ ” and “ \bullet ”, with $*$ = (\prec) + (\succ) + (\bullet). Burgunder and Ronco

applied a similar ternary splitting to surjections. Such a ternary splitting yields an algebraic structure known as a tridendriform algebra (or more generally, q -tridendriform algebra).

Associahedra and permutohedra are specific instances of a much broader class of polytopes called hypergraph polytopes (or nestohedra), which are geometrically obtained by truncating the faces of simplices. Recent work by Curien, Ivanović and Obradović showed that the faces of any hypergraph polytope can be described via tree-like structures called constructs. This construct-based syntax provides the foundational framework to generalize the classic tree and surjection shuffles to other families of polytopes.

Going beyond traditional binary operations, recent work of Curien, Delcroix-Oger and Obradović transitioned this framework into an “unbiased” setting where operations take an arbitrary number of arguments. This gives rise to a novel algebraic structure called a polydendriform algebra, from which standard tridendriform equations can be elegantly synthesized.

The goal of this internship is to:

- get familiar with the structural conditions that permit the extension of the classical shuffle product to various families of hypergraph polytopes,
- get familiar the polydendriform equations of the resulting shuffle products and explore their algebraic properties,
- investigate the order-theoretic and non-recursive characterization of the resulting shuffle products.

Prerequisites: This internship does not require extensive specialized background knowledge. A solid foundation in discrete mathematics, basic combinatorics, and algebra, together with the ability to read mathematical texts in English, is sufficient.

Relevant references:

- 1 Jean-Louis Loday and María O. Ronco. “Hopf algebra of the planar binary trees”. English. In: Adv. Math. 139.2 (1998), pp. 293–309.
- 2 Emily Burgunder and María Ronco. “Tridendriform structure on combinatorial Hopf algebras.” English. In: J. Algebra 324.10 (2010), pp. 2860–2883.
- 3 Pierre-Louis Curien, Jovana Obradović, and Jelena Ivanović. “Syntactic aspects of hypergraph polytopes”. English. In: J. Homotopy Relat. Struct. 14.1 (2019), pp. 235–279.
- 4 Pierre-Louis Curien, Bérénice Delcroix-Oger, and Jovana Obradović. “Tridendriform algebras on hypergraph polytopes”. en. In: Algebraic Combinatorics 8.1 (2025), pp. 201-234.
- 5 Pierre-Louis Curien, Bérénice Delcroix-Oger, and Jovana Obradović. “Tridendriform algebras on hypergraph polytopes, the other way around”. preprint. arXiv:2606.17755. (2026).

6 Pierre-Louis Curien, Bérénice Delcroix-Oger, and Jovana Obradović. “Generalised flip order on the faces of nestohedra”. preprint. (2026).

Research in Functional Analysis

Mentors:

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Nebojša Dinčić, nebojsa.dincic@pmf.edu.rs

Description: Functional analysis occupies the central part of mathematical analysis. The results from this area have fundamental implications in understanding the core processes and problems in differential and partial differential equations, harmonic analysis, numerical analysis, stochastics, linear algebra, matrix analysis, and, since recently, in machine learning and neural networks.

Within this topic, the research would be adapted to the candidate: candidate’s prior knowledge and affinities would be taken into account, and accordingly the application of functional analysis in some of the above-mentioned areas would be investigated.

Prerequisites: Basic courses in algebra and linear algebra, matrix analysis, real analysis, complex analysis, measure theory and integration, functional analysis and operator theory, and the ability to read and understand mathematical texts in English.

Available in Serbian, English, and French.

Deep Structure of the Image

Mentor: Miloš Milovanović, milosm@turing.mi.sanu.ac.rs

Description: The deep structure of the image in signal processing implies a scaling parameter which determines the signal supplementing spatial coordinates. It represents the resolution and dynamics establishing the intrinsic time of a system. The structure is followed concurrently in spatial art such as sculpture and architecture, as well as spatial arts such as music. Its consideration allows for a general definition of time that should relate to the growth of information, regardless of a representative medium.

Prerequisites: Background knowledge is not particularly necessary, but should relate primarily to the differential geometry of smooth manifolds and signal processing. A programming language is also useful, preferably Matlab with the wavelet toolbox.

Relevant references:

- 1 Florack, L. Image Structure; Springer, Heidelberg, 2010.
- 2 Milovanović, M. & Saulig, N. The Duality of Psychological and Intrinsic Time in Artworks, *Mathematics* 2024, 12(12), 1850.
- 3 Milovanović, M. & Tomić, B.M. Fractality and Self-organization in the Orthodox Iconography, *Complexity* 2016, 21(S1), 55—68.

Quantum Probability Theory

Mentors:

Miloš Milovanović, milosm@turing.mi.sanu.ac.rs

Borivoje Dakić

Description: Along with the classical probability established by Andrei Kolmogorov, quantum theory emerged in the opus of John von Neumann. Their relationship represents the opposition of the extensional view based upon classical set logic and the intensional which refers to properties presented by orthoprojectors. The goal of the research is to develop the approach into a consistent theory that should be a counterpart to the classical one and encompass it.

Prerequisites: Knowledge of probability at least on the level of Mathematical Gymnasium is required. Elementary knowledge of quantum physics should also be desirable. Basic knowledge of mathematical analysis and geometry is assumed.

Relevant references:

- 1 Khrennikov, A. Probability and Randomness: Quantum versus Classical, Imperial College Press, 2016.
- 2 Milovanović, M. & Saulig, N. An Intensional Probability Theory: Investigating the Link between Classical and Quantum Probabilities, *Mathematics* 2022, 10(22), 4294.
- 3 Pitowsky, I. Quantum Probability — Quantum Logic, Springer-Verlag, Berlin Heidelberg, 1989.

4 Wojciechowski, B.W. Quantum Probability Theory, Psychology and Law: Modelling Legal Decision Making with Quantum Principles, Routledge, London, 2023.

The Black Box Problem in the Justification of Formal Decisions

Mentors:

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Description: The application of artificial intelligence in systems that require the justification of formal decisions, such as the judiciary and banking sectors, confronts legal theory with a challenge known as the black box problem. This topic examines the relationship between the technical opacity of modern machine learning algorithms and the legal requirement that every decision producing legal effects must be transparent and properly justified. The aim of the research is to establish algorithmic and legal approaches for ensuring transparency, accountability, and legal certainty in addressing the black box problem.

Prerequisites: Artificial intelligence, neural networks, large language models, and legal studies.

Relevant references:

- 1 Russell, S., & Norvig, P. (2020). *Artificial Intelligence: A Modern Approach* (4th ed.). Pearson.
- 2 Lipton, Z. C. (2018). The mythos of model interpretability. *Communications of the ACM*, 61(10), 36–43. <https://doi.org/10.1145/3233231>
- 3 Wachter, S., Mittelstadt, B., & Russell, C. (2017). Counterfactual explanations without opening the black box: Automated decisions and the GDPR. *Harvard Journal of Law & Technology*, 31(2), 841–887.
- 4 Rudin, C. (2019). Stop explaining black box machine learning models for high stakes decisions and use interpretable models instead. *Nature Machine Intelligence*, 1(5), 206–215. <https://doi.org/10.1038/s42256-019-0048-x>